

Dispersive properties and giant Kerr non-linearities in Dipole Induced Transparency

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Abstract

We calculate the dispersive properties of the reflected field from a cavity coupled to a single dipole. We show that when a field is resonant with the dipole it experiences a π phase shift relative to reflection from a bare cavity if the Purcell factor exceeds the bare cavity reflectivity. We then show that optically Stark shifting the dipole with a second field can be used to achieve giant Kerr non-linearities. It is shown that currently achievable cavity lifetimes and cavity quality factors can allow a single emitter in the cavity to impose a nonlinear π phase shift at the single photon level.

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Optical non-linearities play an important role in quantum optics, quantum information processing, and also in design of practical quantum electronic devices. One of the most commonly addressed non-linearities is the Kerr effect which can result in cross-phase modulation and two-photon absorption. These effects have a large number of applications for optical detection, all-optical switching, and quantum computation [1, 2]. The main difficulty in achieving these applications is that conventional materials offer only a very small non-linear response, which is significantly outweighed by linear absorption. Furthermore, applications in quantum optics and quantum information often require that these non-linearities be created by a small number of photons, or sometimes even a single photon. There are very few situations where one can even approach this regime.

One of the few cases where Kerr non-linearities with a small number of photons can be observed is in atomic vapors using Electromagnetically Induced Transparency (EIT) [3]. Due to the large atomic coherence of these systems, cross phase modulation and two-photon absorption can be observed with only a small number of photons [4]. The central idea for these schemes was originally presented by Schmidt and Imamoglu [5]. This scheme exploits EIT in a 4-state atom where the interaction between two weak pulses is mediated by a strong coupling laser. A limitations of the Schmidt-Imamoglu proposal is that large group velocity mismatch between the two interacting pulses puts a fundamental lower limit on the required intensity for a full π phase shift to a few photons per cubic wavelength [6].

In this paper we explore the dispersive properties of light that is reflected from a cavity containing a single dipole emitter. We calculate absorptive and dispersive properties of the reflection coefficient as a function of the coupling strength between the dipole and cavity. We show that for large values of the Purcell factor, losses due to cavity leakage and dipole absorption are cancelled. This is a manifestation of destructive interference which inhibits the light from entering the cavity, and is known as Dipole Induced Transparency (DIT) [7]. At the same time, when the Purcell factor exceeds the bare cavity reflectivity, the presence of a dipole imposes a 0 phase shift on the reflected field that is resonant with the dipole frequency, whereas a bare cavity would impose a π phase shift. This change of phase has been previously studied in the strong coupling regime of cavity QED [10]. Our result shows that only weak coupling is required to observe this dipole induced phase shift. Thus, DIT provides a special condition where we can drive the dipole on resonance and create large phase shifts while not suffering from absorption.

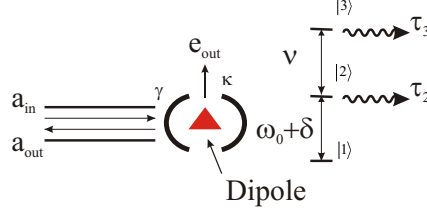


FIG. 1: Setup for large Kerr non-linearities off of reflection from a cavity coupled to a single dipole.

The large dipole induced phase shifts in DIT are sharply peaked near the resonant frequency of the emitter, creating a highly dispersive region with large group delays. This large dispersion allows us to create giant Kerr non-linearities by optically Stark shifting the dipole. This method of achieving non-linearities is significantly easier to implement than the Schmidt-Imamoglu scheme using EIT. Instead of requiring a 4-level system, we only require a 3-level system for the dipole. When using a quantum dot emitter, these levels can be readily provided by the single exciton and bi-excitonic resonance. Furthermore, we do not need an external coupling laser, since coupling is provided by the vacuum Rabi frequency of the dipole system. Finally, our proposal can exploit the large Rabi frequencies provided by photonic crystal micro-cavities to achieve non-linearities with only a single emitter. We show that with presently achievable cavity lifetimes and vacuum Rabi frequencies, working only in the weak coupling regime, we can achieve a full non-linear π phase shift using only a single emitter and a single photon in the cavity.

The system we study in this paper is shown in Fig. 1. An external waveguide field is reflected off of a single-sided cavity containing a dipole. We define γ as the energy coupling rate from cavity to waveguide, while κ is the energy coupling rate into parasitic leaky modes. The dipole has three states, denoted $|1\rangle$, $|2\rangle$, and $|3\rangle$, with transition frequencies $\omega_0 + \delta$ and ν , where ω_0 denotes the central frequency of the cavity. We assume that $\delta, \omega_0 - \nu < \gamma + \kappa$, so that both transitions can couple to the cavity mode. We denote σ_-^{12} and σ_-^{23} as the lowering operators for the dipole, and $\hat{\mathbf{b}}$ as the bosonic annihilation operator for the cavity field.

The Hamiltonian of the system is given by

$$H = H_s + \sum_{n=1}^N \hbar \left[g_1 \hat{\mathbf{b}}^\dagger \sigma_-^{12} + g_2 \left(\hat{\mathbf{b}} + \hat{\mathbf{b}}^\dagger \right) \sigma_-^{23} + \text{H.c.} \right] \quad (1)$$

where g_1 and g_2 are the vacuum rabi frequencies for the 1-2 and 2-3 transitions respectively. H_s is the Hamiltonian of the uncoupled systems and the cavity-waveguide interaction terms.

In the first part of the paper we only consider the 1-2 transition to calculate the dispersive properties of the cavity in the presence of a dipole. Thus, we set $g_2 = 0$. In the second part of the paper we add the 2-3 transition and consider the case where the dipole population is mainly in state $|1\rangle$. Thus, the 2-3 transition does not drive the cavity but can still create an optical Stark shift on state $|2\rangle$. In order to properly derive the Stark shift term, we do not yet make the rotating wave approximation for the interaction between the cavity and σ_-^{23} .

Using the above Hamiltonian, along with standard cavity input-output formalism [8], the Heisenberg picture equations of motion for the operators are given by

$$\frac{d\hat{\mathbf{b}}}{dt} = (-i\omega_0 + \gamma/2 + \kappa/2)\hat{\mathbf{b}} - \sqrt{\gamma}\hat{\mathbf{a}}_{out} - \sqrt{\kappa}\hat{\mathbf{e}}_{out} - ig_1\sigma_-^{12} \quad (2)$$

$$\frac{d\sigma_-^{12}}{dt} = \left(-i(\omega_0 + \delta) + \frac{\tau_2}{2}\right)\sigma_-^{12} - ig_1\hat{\mathbf{b}} - ig_2(\hat{\mathbf{b}} + \hat{\mathbf{b}}^\dagger)\sigma_-^{12}\sigma_-^{23} + \hat{\mathbf{f}} \quad (3)$$

$$\frac{d\sigma_-^{23}}{dt} = \left(-i\nu + \frac{\tau_3}{2}\right)\sigma_-^{23} - ig_2(\hat{\mathbf{b}} + \hat{\mathbf{b}}^\dagger)\sigma_z^{23} + \hat{\mathbf{h}} \quad (4)$$

The operators $\hat{\mathbf{f}}$ and $\hat{\mathbf{h}}$ are noise operators that are needed to conserve the commutation relation of the dipole operators. The above equations are derived with the assumption that there is virtually no population in the states $|2\rangle$ and $|3\rangle$, so σ_z^{12} and σ_z^{23} are time invariant, and σ_-^{23} does not drive the cavity mode. The condition for this assumption to be valid is given by $\langle\sigma_+\sigma_-\rangle \ll 1$, which is equivalent to the condition $\langle\hat{\mathbf{a}}_{in}^\dagger\hat{\mathbf{a}}_{in}\rangle \ll g_1^2/\gamma$ for an input field that is resonant with the dipole [7]. This condition is well satisfied in the operating regime we consider. The input and output fields are related by

$$\hat{\mathbf{a}}_{out} - \hat{\mathbf{a}}_{in} = \sqrt{\gamma}\hat{\mathbf{b}} \quad (5)$$

We begin by first studying the dispersive properties of reflected light when we only have the transition σ_-^{12} and set $g_2 = 0$. In this case, if we apply an input field at frequency ω , the reflection coefficient can be solved using Eq. 2-5 and is given by

$$r(\omega) = \frac{i\Delta\omega + \frac{g_1^2}{i(\Delta\omega+\delta)+\tau_2/2} - \gamma/2 + \kappa/2}{i\Delta\omega + \frac{g_1^2}{i(\Delta\omega+\delta)+\tau_2/2} + \gamma/2 + \kappa/2} \quad (6)$$

We define the amplitude and phase of the reflection coefficient by $r = \sqrt{R}e^{i\Phi_r}$, where R is the reflectivity of the cavity and Φ_r is the phase shift imposed on the reflected field.

Fig. 2 plots the cavity reflectivity as a function of detuning from the cavity resonance for several values of g_1 . We set $\gamma = 6THz$ and $\kappa = 0.1THz$, which is the decay rate of a cavity

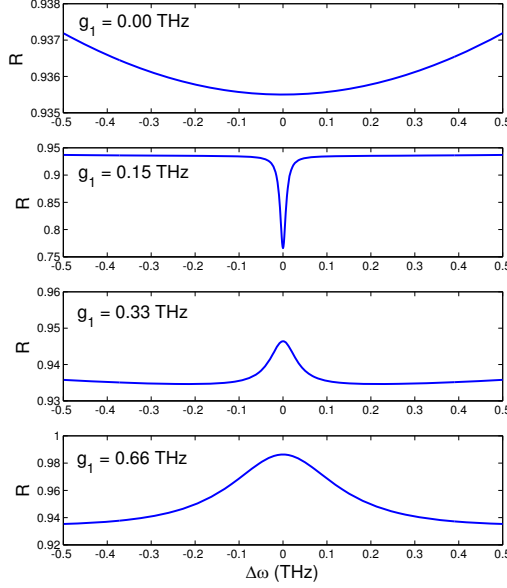


FIG. 2: Cavity reflection coefficient for different values of g .

with a quality factor of $Q = 10,000$ (here Q is defined as $Q = \omega_0/\kappa$). The dipole is assumed to be resonant with the center frequency of the cavity so that $\delta = 0$, while $\tau_2 = 1GHz$, a value taken from experimental measurements on quantum dots [9].

When $g_1 = 0$ the cavity is not perfectly reflecting due to the coupling to leaky modes given by κ . Introducing a small g_1 increases the cavity loss on-resonance, because the dipole behaves as an absorber. However, when g_1 is increased to higher values, the cavity reflection improves. This can be understood from the reflection coefficient at $\Delta\omega = 0$, which is given by

$$r(\omega_0) = (F_p - r_0)/(F_p + 1) \quad (7)$$

where $F_p = 4g^2/[\tau_2(\gamma + \kappa)]$ is known as the Purcell factor, and $r_0 = (\gamma - \kappa)/(\gamma + \kappa)$ is the reflection coefficient for a bare cavity with no dipole. When $F_p \approx r_0$, the cavity is very lossy because most of the field is absorbed by the dipole. However, when $F_p \gg r_0$ the cavity becomes very reflective. This is the result of destructive interference of the cavity mode when it is coupled to the dipole, which prevents light from entering the cavity and therefore inhibits losses [7].

We next plot Φ_r , normalized by π , in Fig. 3 for different values of g_1 . When $g_1 = 0$ the field experiences a π phase shift on reflection, which is the expected behavior for reflection off of a bare cavity. For nonzero g_1 , we see significant modification of the reflection phase

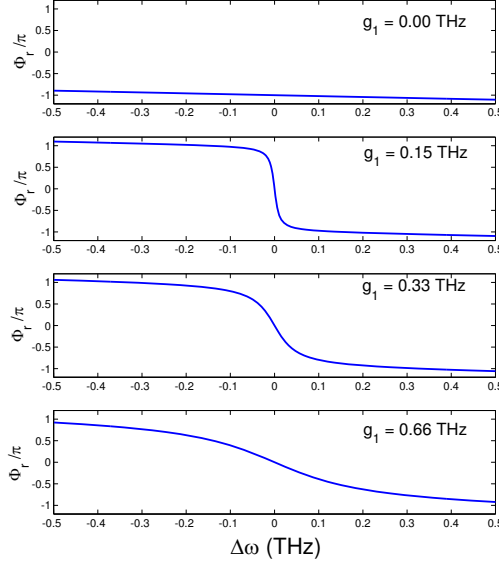


FIG. 3: Phase shift after reflection from cavity for several values of g .

near the dipole resonance. Instead of a π phase shift, we now have a 0 phase shift. This shift quickly changes to π at a detuning of $\Delta\omega \approx g_1^2/(\gamma/2 + \kappa/2)$. This phase change can be easily understood from Eq. 7, which shows that when $F_p > r_0$ the reflection coefficient is positive, while $F_p < r_0$ implies a negative reflection coefficient.

It is significant to note that the dipole imposes a large change of phase whenever $F_p > r_0$. In previous work by Duan and Kimble [10], it has been shown that this change in reflection phase can be used to perform a controlled phase gate by performing successive reflections of two photons off of a cavity mode. In that proposal, the authors speculated that the strong coupling regime is required to achieve proper performance. Here we show that strong coupling is not required for implementation of the Duan-Kimble proposal. A Purcell factor greater than r_0 already achieves this desired phase shift. We need only create sufficiently large values for F_p so that the cavity is not lossy.

The sharp dispersive feature of the reflection coefficient also opens up the possibility to achieve large Kerr non-linearity. If we can shift the dispersion curve by a very small amount, on the order of $\Delta\omega \approx g_1^2/(\gamma/2 + \kappa/2)$, we can change the reflection phase from π to 0. The shift in dispersion can be created by optically Stark shifting the 1-2 transition by applying an off-resonant field on the 2-3 transition.

To calculate the optical Stark shift, we assume the input field has two frequency components, one at ω and the other at $\nu + \Delta$. The component at $\nu + \Delta$ is the field responsible for

creating a Stark shift. The response of the cavity at this frequency is given by substituting $\hat{\mathbf{a}}_{in} = \hat{\mathbf{a}}_{\nu+\Delta} e^{-i(\nu+\Delta)t}$ and taking the Fourier transform of Eq. 2 at frequency $\nu + \Delta$. This gives us

$$\hat{\mathbf{b}}_{\nu+\Delta} = \frac{-\sqrt{\gamma}\hat{\mathbf{a}}_{\nu+\Delta}}{i(\omega_0 - \nu - \Delta) + \gamma} \approx \frac{-\hat{\mathbf{a}}_{\nu+\Delta}}{\sqrt{\gamma}} \quad (8)$$

We now assume that σ_-^{23} is driven mainly by the field component at frequency $\nu + \Delta$. Using this approximation, we calculate this Fourier component of Eq. 4, which is given by

$$\sigma_-^{23}(\nu + \Delta) = \frac{-ig_2\sigma_z^{23}(\hat{\mathbf{b}}_{\nu+\Delta} + \hat{\mathbf{b}}_{\nu+\Delta}^\dagger)}{i\Delta + \frac{\tau_3}{2}} \quad (9)$$

We substitute the above expression back into Eq. 3 and make the rotating wave approximation, which gives us

$$\frac{d\sigma_-^{12}}{dt} = \left(-i(\omega_0 + \delta) + \frac{\tau_2}{2} - i\hat{\mathbf{S}}\right)\sigma_-^{12} + ig_1\sigma_z^{12}\hat{\mathbf{b}} + \sqrt{\tau_3}\hat{\mathbf{f}} \quad (10)$$

where

$$\hat{\mathbf{S}} = \frac{i2g_2^2\hat{\mathbf{b}}_{\nu+\Delta}^\dagger\hat{\mathbf{b}}_{\nu+\Delta}}{i\Delta + \frac{\tau_3}{2}} \quad (11)$$

The Stark operator $\hat{\mathbf{S}}$ has a real and imaginary component. The real component gives the optical Stark shift, while the imaginary component represents loss due to two-photon absorption. When $\Delta \gg \tau_3$ this operator represents an energy shift that is proportional to the number of photons at frequency $\nu + \Delta$. If, instead, the field is on resonance with the transition from $|2\rangle$ to $|3\rangle$ so that $\Delta \rightarrow 0$, the Stark operator becomes a loss coefficient. When pumped by a bright field $\hat{\mathbf{a}}_{\nu+\Delta}$, we can substitute $\hat{\mathbf{b}}_{\nu+\Delta}^\dagger\hat{\mathbf{b}} = \langle\hat{\mathbf{a}}_{\nu+\Delta}^\dagger\hat{\mathbf{a}}_{\nu+\Delta}\rangle/\gamma$.

Using the above expressions, we can once again solve for the cavity reflection coefficient when the cavity is driven by a monochromatic input at frequency ω . The reflection coefficient is now given by

$$r(\omega) = \frac{i\Delta\omega + \frac{g_1^2}{i(\Delta\omega + \delta + \hat{\mathbf{S}}) + \tau_2} - \gamma/2 + \kappa/2}{i\Delta\omega + \frac{g_1^2}{i(\Delta\omega + \delta + \hat{\mathbf{S}}) + \tau_2} + \gamma/2 + \kappa/2} \quad (12)$$

which is nearly identical to the $g_2 = 0$ case, except that the dipole is now detuned by an additional value determined by $\hat{\mathbf{S}}$.

We consider the case where there is exactly one photon in the cavity at frequency $\nu + \Delta$, in which case we can substitute $\hat{\mathbf{b}}_{\nu+\Delta}^\dagger\hat{\mathbf{b}}_{\nu+\Delta} = 1$, and $\Delta \gg \tau_3$ so that the Stark operator causes only a phase shift. To calculate the Stark shift we must have a value for g_2 . In the case of a quantum dot where we use the single and bi-exciton transitions as the two

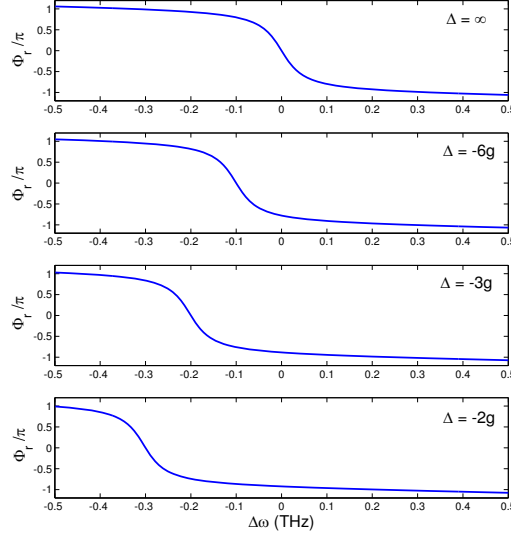


FIG. 4: Reflection phase shift in presence of a second field at frequency $\nu + \Delta$ for several values of Δ . We set $g = 0.3THz$.

excited states, it is reasonable to assume that $g_2 = g_1 = g$. This is because both transitions represent an absorption of a photon by an exciton in the quantum dot. For other systems, the value of g_2 must be measured or calculated from the matrix element between $|2\rangle$ and $|3\rangle$.

Making the assumption that $g_1 = g_2 = g$, Fig. 4 plots the phase of the reflection coefficient for several values of Δ , with $g = 0.3THz$. We first plot the case of $\Delta = \infty$, in which case there is no optical Stark shift. The curve is therefore identical to the one in Fig. 3. As we bring the field closer to resonance with the second transition of the dipole, the resonant frequency of the first transition is shifted by $2g^2/\Delta$. The phase shift curve will therefore be translated to the new resonant frequency of the dipole. As the figure shows, $\Delta \approx -6g$ is enough to change the reflection phase from 0 to π in the presence of a single photon. In general, when $|\Delta| = \kappa/2 + \gamma/2$ we have a frequency shift of $2g^2/(\kappa + \gamma)$, which is sufficiently large to change the phase by π . This means that any field which is resonant with the cavity can provide a π phase shift.

The system we consider exhibits large Kerr non-linearities due to two properties. The first is that the confinement of the cavity field creates large values of g , which in turn generate very strong optical Stark shifts. But there is a second, more critical property. The largest phase difference is experienced near resonance with the dipole. In a normal system, we

would not be able to drive a dipole resonantly without simultaneously suffering from large losses. In the case of a DIT, however, we can drive the system on resonance and not suffer from absorption. As Fig. 2 shows, the losses are smallest when the input field is resonant with the cavity. This is because when we drive the system on resonance, the cavity field destructively interferes and thus inhibits any external driving field from entering the cavity. This eliminates absorption, allowing us to work in a regime where we can go on resonance while not suffering from optical losses.

In conclusion, we have calculated the dispersive properties of reflection from a cavity seeded with a single emitted, and shown that when $F_p > r_0$, where r_0 is the bare cavity reflectivity, a 0 phase shift is induced on the reflected field, instead of a π phase shift for a bare cavity. We showed that by optically Stark shifting the dipole, we can create large cross-phase modulation angles at the single photon level.

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